Webology (ISSN: 1735-188X)
Volume 18, Number 1, 2021

THE α-, β-, γ- DUALS OF THE SPACES $bs^u(T)$, $cs^u(T)$ AND $cs_0^u(T)$AND ITS MATRIX TRANSFORMATIONS

B. ESSAKIAMMAL @ SHUNMUGAM[1] & A. Pandiarani[2]

[1]Research Scholar, Reg No: 20122052092003, PG and Research Department of Mathematics, G. Venkataswamy Naidu College, Kovilpatti-628 502, Tamil Nadu, India, Affliation of Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627 012, Tamil Nadu, India.

[2]Associate Professor, PG and Research Department of Mathematics, G. Venkataswamy Naidu College, KOvilpatti-628 502, Tamil Nadu, India, Affliation of Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627 012, Tamil Nadu, India.

Abstract

We have computed the α−, β− and γ− duals of the new sequence spaces $bs^u(T)$, $cs^u(T)$ and $cs_0^u(T)$ of non-absolute type which have recently been introduced. Also, we Characterize some matrix classes from the spaces $bs^u(T)$, $cs^u(T)$ and $cs_0^u(T)$ to the spaces l_p, c and c_0, where $1 \le p \le \infty$.

Keywords: α−, β− and γ− duals, matrix mappings, transformations.

1. Introduction:

Let ω be the space of all real valued sequences. A sequence space is a vector space whose elements are infinite sequences of real or complex numbers. For instance, the classical sequence spaces l_∞, c and c_0 are BK- spaces with their usual norms, are the spaces of all bounded, convergent and null sequences respectively normed by $|x| = \sup_k |x_k|$. Also, bs, cs, l_1 and l_p with p-norm $\| x \|_p = (\sum_{k=1}^{\infty} |x_k|^p)^{1/p}$ denote the spaces of all bounded, convergent, absolutely and p-absolutely convergent series respectively, where $1 \le p \le \infty$. That is,

$bs = \{x \in w : (\sum_{k=1}^{n} x_k) \in l_\infty\}$: $cs = \{x \in w : (\sum_{k=1}^{n} x_k) \in c\}$ and $cs_0 = \{x \in w : (\sum_{k=1}^{n} x_k) \in c_0\}$.

Let X and Y be any two Sequence spaces and $A = (a_{nk})$ be an infinite matrix of real numbers a_{nk}, where n, k ∈ N. Then we say that A defines a matrix transformation from X into Y and is denoted by A: X → Y if for every sequence $x = (x_k) \in X$, the sequence $Ax = (A_n(x))$, the A-transform of x is in Y where $A_n(x) = \sum_k a_{nk} x_k$ (k, n ∈ N). (1.1)

For simplicity in notion here and in what follows, the summation without limit runs from 0 to ∞. By $\mathcal{F}$, we shall denote the collection of all finite subsets of N. By (X, Y), we denote the class of all matrices A such that A: X→ Y. Thus, A ∈ (X, Y) if and only if the series on the right hand side of (1.1) converges for each n ∈ N and every x ∈ X, we have $Ax = \{A_n(x)\}_{n\in N} \in Y$ for all $x \in X$.

Assume here and after that $u = (u_k)$ be an sequence such that $u_k \neq 0$ for all $k \in N$ and $t = (t_k)$ be a strictly increasing sequence of positive reals. By using the matrix domain of a triangle infinite matrix many sequence spaces have recently been defined by several authors. The new Generalized difference triangle matrix $T = (t^u_{nk})$ constituted by using sequence (t_n) and non-zero sequence (u_n) have been introduced and studied by Shanmugavel and Pandiarani[15] to the sequence spaces $c^u_0(T), c^u(T), l^u_\infty(T)$ and $l^u_p(T)$ of non-absolute type. By using the matrix domain T introduced by Shanmugavel and Pandiarani[15], Essakiammal @ Shunmugam and Pandiarani[6] have just introduce the new sequence spaces $bs^u(T)$, $cs^u(T)$ and $cs^u_0(T)$ of non-absolute type which are the set of sequences whose T-transform are in the spaces bs, cs and cs_0 respectively, where T denotes the matrix $T = (t^u_{nk})$ defined by

$$t^u_{nk} = \begin{cases} \frac{t_n - t_{n-1}}{t_n} u_n & \text{if } k = n \\ \frac{t_{n-2} - t_{n-1}}{t_n} u_{n-1} & \text{if } k = n-1 \\ 0 & \text{if } 0 \le k < n-1 \\ & \quad \text{(or) } k > n \quad (n, k \in N) \end{cases}$$

Following Choudhary and Mishra[5], Basar and Altay[1][2][3], Shanmugavel and Pandiarani[15], the new sequence spaces $bs^u(T)$, $cs^u(T)$ and $cs^u_0(T)$ which were defined by Essakiammal @ Shunmugam and Pandiarani [6]as follows:

$$bs^u(T) = \{x = (x_k) \in \omega : \operatorname{Sup}_{k\in N} |\textstyle\sum_{n=1}^{k} \left(\frac{t_n - t_{n-1}}{t_n} u_n x_n - \frac{t_{n-1} - t_{n-2}}{t_n} u_{n-1} x_{n-1}\right)| < \infty\}$$

$$cs^u(T) = \{x = (x_k) \in \omega : \lim_{k\to\infty} \textstyle\sum_{n=1}^{k} \left(\frac{t_n - t_{n-1}}{t_n} u_n x_n - \frac{t_{n-1} - t_{n-2}}{t_n} u_{n-1} x_{n-1}\right) \text{ exists}\}$$

$$cs^u_0(T) = \{x = (x_k) \in \omega : \lim_{k\to\infty} \textstyle\sum_{n=1}^{k} \left(\frac{t_n - t_{n-1}}{t_n} u_n x_n - \frac{t_{n-1} - t_{n-2}}{t_n} u_{n-1} x_{n-1}\right) = 0\}$$

Define the sequence $y = (y_n)$ which will be frequently used as the T-transforms of a sequence $x = (x_n)$, i.e:) $y_n = T_n(x) = \frac{t_n - t_{n-1}}{t_n} u_n x_n - \frac{t_{n-1} - t_{n-2}}{t_n} u_{n-1} x_{n-1}$ $(n \in N)$ (1.2)

2. The α-, β-, γ- duals of the spaces $bs^u(T), cs^u(T)$ and $cs^u_0(T)$

In this present section, we have computed the α-, β-, γ- duals of the spaces $bs^u(T), cs^u(T)$ and $cs^u_0(T)$. For arbitrary spaces X and Y, the set M(X, Y) defined by

$$M(X, Y) = \{a = (a_k) \in \omega : ax = (a_k x_k) \in Y \text{ for all } x = (x_k) \in X\} \quad (2.1)$$

is called the multiplier space of X and Y. It is easy to observe that the sequence space Z with Y⊂ Z and Z ⊂ X that the inclusion M(X, Y) ⊂ M(X, Z) and M(X, Y) ⊂ M(Z, Y) hold respectively.

With the notation of (2.1), the $\alpha-$, $\beta-$ and $\gamma-$ duals of a sequence space X respectively are denoted by X^{α}, X^{β} and X^{γ} are defined by

$X^{\alpha} = M(X, l_1)$

$X^{\beta} = M(X, cs)$

$X^{\gamma} = M(X, bs)$

It is clear that $X^{\alpha} \subset X^{\beta} \subset X^{\gamma}$. It is obvious that the inclusions

$X^{\alpha} \subset Y^{\alpha}$, $X^{\beta} \subset Y^{\beta}$ and $X^{\gamma} \subset Y^{\gamma}$ hold, whenever $Y \subset X$.

The following are the known fundamental results for this section.

Lemma 2.1 $A = (a_{nk}) \in (cs: l_1)$ if and only if

$$\mathrm{Sup}_{N,K \in \mathcal{F}} \left| \sum_{n \in N} \sum_{k \in K} (a_{nk} - a_{n,k-1}) \right| < \infty \tag{2.2}$$

Lemma 2.2 $A = (a_{nk}) \in (cs_0: l_1)$ if and only if

$$\mathrm{Sup}_{N,K \in \mathcal{F}} \left| \sum_{n \in N} \sum_{k \in K} (a_{nk} - a_{n,k+1}) \right| < \infty. \tag{2.3}$$

Lemma 2.3 $A = (a_{nk}) \in (bs: l_1)$ if and only if (2.3) holds and

$$\lim_{k} a_{nk} = 0 \quad \forall\, n \in N. \tag{2.4}$$

Lemma 2.4 $A = (a_{nk}) \in (cs: c)$ if and only if

$$\mathrm{Sup}_n \sum_k \left| a_{nk} - a_{n,k+1} \right| < \infty \text{ and} \tag{2.5}$$

$$\lim_{k} a_{nk} \text{ exist for all } k \in N. \tag{2.6}$$

Lemma 2.5 $A = (a_{nk}) \in (cs_0: c)$ if and only if (2.5) holds and

$$\lim_{n} (a_{nk} - a_{n,k+1}) \text{ exist for all } k \in N. \tag{2.7}$$

Lemma 2.6 $A = (a_{nk}) \in (bs: c)$ iff (2.4) and (2.6) holds and

$$\sum_k \left| a_{nk} - a_{n,k-1} \right| \text{ converges.} \tag{2.8}$$

Lemma 2.7 $A = (a_{nk}) \in (cs: l_\infty)$ if and only if

$$Sup_n \sum_k \left| a_{nk} - a_{n,k-1} \right| < \infty. \tag{2.9}$$

Lemma 2.8 $A = (a_{nk}) \in (cs_0: l_\infty)$ if and only if (2.5) holds.

Lemma 2.9 $A = (a_{nk}) \in (bs: l_\infty)$ iff (2.4) and (2.5) holds.

Theorem 2.10 Define the sets M_1 and M_2 as follows

M_1= {a= $(a_n) \in \omega$: $Sup_{N,K \in \mathcal{F}}\left|\sum_{n\in N}\sum_{k \in K}(v_{nk} - v_{n,k-1})\right| < \infty$

M_2= {a= $(a_n) \in \omega$: $Sup_{N,K \in \mathcal{F}}\left|\sum_{n\in N}\sum_{k \in K}(v_{nk} - v_{n,k+1})\right| < \infty$, where the matrix V = v_{nk} is defined via the sequence a= $(a_n) \in \omega$ by

$$v_{nk}=\begin{cases}\frac{t_k}{(t_n-t_{n-1})u_n}a_n & 0 \le k \le n \\ 0 & k > n\end{cases}$$

for all n, k ∈ N. Then $\{cs^u(T)\}^\alpha = M_1$ and $\{cs_0^u(T)\}^\alpha = \{bs^u(T)\}^\alpha = M_2$

Proof: Let a= $(a_n) \in \omega$. Then by considering the relation

$y_n = T_n(x) = \frac{t_n-t_{n-1}}{t_n}u_nx_n - \frac{t_{n-1}-t_{n-2}}{t_n}u_{n-1}x_{n-1}$ (n ∈ N) and $x_k = \sum_{j=0}^{k}\frac{t_j}{(t_k-t_{k-1})u_k}y_j$, we get an immediate equality that $a_nx_n = \sum_{k=0}^{n}\frac{t_k}{(t_n-t_{n-1})u_k}a_ny_k = V_n(y)$, n ∈ N. (2.10)

From (2.10), we observe that ax = $(a_nx_n) \in l_1$, whenever x = $(x_k) \in cs^u(T)$ iff $V_n(y) \in l_1$, whenever y = $(y_k) \in cs$.

From this, it means that the sequence a = $(a_n) \in \{cs^u(T)\}^\alpha$ if and only if V ∈ $(cs: l_1)$.

Hence, we obtain by lemma 2.1 with V instead of A that a = $(a_n) \in \{cs^u(T)\}^\alpha$ if and only if $Sup_{N,K \in \mathcal{F}}\left|\sum_{n\in N}\sum_{k \in K}(v_{nk} - v_{n,k-1})\right| < \infty$ this yields that $\{cs^u(T)\}^\alpha = M_1$.

Similarly, we can deduce from lemma 2.3 with (2.10) that a = $(a_n) \in \{bs^u(T)\}^\alpha$ if and only if V ∈ (bs:l_1). Clearly, the columns of the matrix B are in the space c_0 since $\lim_n v_{nk} = 0$ for all k ∈ N.

From (2.3), we obtain that $Sup_{N,K \in \mathcal{F}}\left|\sum_{n\in N}\sum_{k \in K}(v_{nk} - v_{n,k+1})\right| < \infty$ (2.11)

This gives that $\{cs_0^u(T)\}^\alpha = \{bs^u(T)\}^\alpha = M_2$. This completes the proof.

Theorem 2.11 Define the sets M_3 and M_4 as follows

$M_3 = \left\{a = (a_k) \in \omega: \sum_{k=0}^{\infty}\left|\tilde{\Delta}\left(\tilde{\Delta}\left(\frac{a_k}{t_k-t_{k-1}}\right)t_k\right)\right| < \infty\right\}$

$M_4 = \left\{a = (a_k) \in \omega: Sup_k\left|\frac{t_k}{t_k - t_{k-1}}a_k\right| < \infty\right\}$

$M_5 = \left\{a = (a_k) \in \omega: lim_{k\to\infty}\left|\frac{t_k}{t_k - t_{k-1}}a_k\right| exists\right\}$

Where $\tilde{\Delta}\left(\frac{a_k}{t_k-t_{k-1}}\right) = \frac{a_k}{t_k-t_{k-1}} - \frac{a_{k+1}}{t_{k+1}-t_k}$ for all k ∈ N. Then $\{cs^u(T)\}^\beta = \{cs_0^u(T)\}^\beta = M_3 \cap M_4$ and $\{bs^u(T)\}^\beta = M_3 \cap M_5$.

Proof: Take any a = $(a_k) \in \omega$ and consider the equation

$$\sum_{k=0}^{n} a_k x_k = \sum_{k=0}^{n}\left[\sum_{j=0}^{k} \frac{t_j}{(t_k - t_{k-1})u_k}\, y_j\right] a_k$$

$$= \sum_{k=0}^{n-1} \tilde{\Delta}\left(\frac{a_k}{t_k - t_{k-1}}\right) t_k y_k + \frac{t_n}{t_n - t_{n-1}} a_n y_n$$

$$= D_n(y) \tag{2.12}$$

Where the matrix D = (D_{nk}^{u}) defined by

$$t_{nk}^{u} = \begin{cases} \tilde{\Delta}\left(\frac{a_k}{t_k - t_{k-1}}\right) t_k & if\ 0 \le k \le n-1 \\ \frac{t_n}{t_n - t_{n-1}}\, a_n & if\ k = n \\ 0 & if\ k > n \end{cases}$$

for all n, k ∈ N.

By (2.12), we deduce that ax = $(a_k x_k) \in cs$ where x = $(x_k) \in cs^{u}(T)$ iff $D_n(y) \in c$ whenever y = $(y_k) \in cs$.

From this, we get a = $(a_k) \in \{cs^{u}(T)\}^{\beta}$ if and only if $D_n(y) \in (cs: c)$.

Therefore, by using lemma 2.4, we derive from (2.5) and (2.6), we have

$\sum_{k=0}^{\infty} \left|\tilde{\Delta}\left(\tilde{\Delta}\left(\frac{a_k}{t_k - t_{k-1}}\right) t_k\right)\right| < \infty,$

$Sup_n \left|\frac{t_k}{t_k - t_{k-1}}\, a_n\right| < \infty$ and

$$\lim_{n} D_{nk}^{u} = \tilde{\Delta}\left(\frac{a_k}{t_k - t_{k-1}}\right) t_k$$

respectively. Thereby , We obtain $\{cs^{u}(T)\}^{\beta} = cs_o^{u}(T)\}^{\beta} = M_3 \cap M_4$

The proof for the space $bs^{u}(T)$ may be obtained in a similar way yields the result that $\{bs^{u}(T)\}^{\beta} = M_3 \cap M_5$

Theorem 2.12 The γ-duals for the spaces $cs^{u}(T)$, $cs_o^{u}(T)$, $bs^{u}(T)$ is the set $M_3 \cap M_4$

Proof: We prove this theorem by using lemma 2.7, lemma 2.8, lemma 2.9 instead of lemma 2.4 in theorem 2.11 we get the desired result .

3. CERTAIN MATRIX MAPPINGS ON THE SPACES $bs^{u}(T), cs^{u}(T)$ *and* $cs_0^{u}(T)$

Here in this section, we characterize the matrix classes $(cs^u(T){:}\ l_p), (cs_0^u(T){:}\ l_p)$, $(bs^u(T){:}\ l_p)$, $(cs^u(T){:}\ c_0)$, $(cs_0^u(T){:}\ c_0)$, $(bs^u(T){:}\ c_0)$, $(cs^u(T){:}\ c)$, $(cs_0^u(T)$:c), $(bs^u(T){:}\ c)$, where 1$\leq p \leq \infty$.

For an infinite matrix A= (a_{nk}) we write that

$$\tilde{a}_{nk}=\tilde{\Delta}\left(\frac{a_{nk}}{\lambda_k-\lambda_{k-1}}\right)\lambda_k = \left(\frac{a_{nk}}{\lambda_k-\lambda_{k-1}}-\frac{a_{n,k+1}}{\lambda_{k+1}-\lambda_k}\right)\lambda_k \qquad (n, k \in N)$$

The list of the following lemmas will be needed in proving our results.

Lemma 3.1 A = $(a_{nk}) \in (cs{:}\ c_0)$ if and only if (2.5) holds and

$$\lim_n a_{nk} = 0\ (\forall\ k \in N) \tag{3.1}$$

Lemma 3.2 A = $(a_{nk}) \in (cs_o{:}\ c_0)$ if and only if (2.5) holds and

$$\lim_n (a_{nk} - a_{n,k+1}) = 0 \qquad (\forall\ k \in N) \tag{3.2}$$

Lemma 3.3 A = $(a_{nk}) \in (bs{:}\ c_0)$ if and only if (2.4) holds and

$$\lim_n \sum_k |a_{nk} - a_{n,k+1}| = 0 \tag{3.3}$$

Lemma 3.4 $A = (a_{nk}) \in (cs_0{:}\ l_p)$ *if and only if*

$$\sup_k \sum_k \left|\sum_{k\in K}(a_{nk} - a_{n,k+1})\right|^p < \infty \qquad (1< p < \infty) \tag{3.4}$$

Lemma 3.5 $A = (a_{nk}) \in (cs{:}\ l_p)$ *if and only if*

$$\sup_k \sum_n \left|\sum_{k\in K}(a_{nk} - a_{n,k-1})\right|^p < \infty \qquad (1< p < \infty) \tag{3.5}$$

Lemma 3.6 $A = (a_{nk}) \in (bs{:}\ l_p)$ *if and only if* (2.4) and (3.4) holds.

Theorem 3.7 (i) $A = (a_{nk}) \in (cs^u(T){:}\ l_\infty)$ if and only if

$$\sup_n \sum_{k=0}^{\infty} |\tilde{a}_{nk} - \tilde{a}_{k-1}| < \infty \quad \text{and} \tag{3.6}$$

$$\sup_k \left|\frac{t_k}{t_k-t_{k-1}} a_{nk}\right| < \infty \tag{3.7}$$

(ii) $A = (a_{nk}) \in (cs_0^u(T){:}\ l_\infty)$ if and only if (3.7) *holds and*

$$\sup_n \sum_{k=0}^{\infty} |\tilde{a}_{nk} - \tilde{a}_{k-1}| < \infty \tag{3.8}$$

(iii) $A = (a_{nk}) \in (bs^u(T){:}\ l_\infty)$ if and only if (3,8) *holds and*

$$\lim_{k\to\infty} \left|\frac{t_k}{t_k-t_{k-1}} a_{nk}\right| \text{ exists,} \tag{3.9}$$

$$\lim_{k} \tilde{a}_{nk} = 0 \text{ and} \tag{3.10}$$

$$sup_n|a_n| < \infty \tag{3.11}$$

Proof: Suppose that the conditions (3.6) and (3.7) holds. Let us take any x = $(x_k) \in cs^u(T)$

Then by theorem 2.11 that $(a_{nk})_{k=0}^{\infty} \in \{cs^u(T)\}^\beta$ for all n $\in N$

From this, the existence of A-transform of x i.e) Ax exists and clearly its associated sequence y = (y_k) is in cs and hence y $\in c_0$. By using the relation of (y_k), the following equality can be derived form the m^{th} partial sum of the series $\sum_k a_{nk}x_k$:

$$\sum_{k=0}^{m} a_{nk}x_k = \sum_{k=0}^{m-1} a_{nk}y_k + \frac{t_m}{t_m - t_{m-1}} a_{nm}y_k \qquad (\forall\, n, m \in N) \tag{3.12}$$

By using (3.6) and (3.7) from (3.12) as m→ ∞. We obtain

$$\sum_k a_{nk}x_k = \sum_k \tilde{a}_{nk}\, y_k \text{ for all n} \in N \tag{3.13}$$

Further, since the matrix $\tilde{A}$ = $(\tilde{a}_{nk})$ is in the class (cs:l_∞) by lemma 3.7 and (3.6) We have $\tilde{A}_y \in l_\infty$.

Thus, by deducing from (1.1) and (3.13) that Ax $\in l_\infty$ and hence A $\in (cs^u(T){:}\ l_\infty)$.

Conversely, Suppose that A $\in (cs^u(T){:}\ l_\infty)$. Then $(a_{nk})_{k=0}^{\infty} \in \{cs^u(T)\}^\beta$ for all n $\in N$.

With theorem 2.11 it implies that both (3.7) and $\sum_{k=0}^{\infty}|\tilde{a}_{nk} - \tilde{a}_{k+1}| < \infty$ for all n $\in N$ and this together imply that the relation (3.13) holds for all sequemces x $\in cs^u(T)$ and y $\in cs$.

By hypothesis , Ax $\in l_\infty$, we obtain by (3.13), that $\tilde{A}_y \in l_\infty$, showing that $\tilde{A} \in (cs{:}\ l_\infty)$ where $\tilde{A} = (\tilde{a}_{nk})$

The necessity of (3.6) is an immediate proof by lemma 2.7. This completes the proof of part (i).

(ii) and (iii) can be proved similarly.

Corollary 3.8 (i) A = $(a_{nk}) \in (cs^u(T){:}\ c)$ if and only if (3.7) and (3.8) hold and

$$\lim_{n\to\infty} (\tilde{a}_{nk}) \text{ exists.} \tag{3.14}$$

(ii) A = $(a_{nk}) \in (cs_o^u(T){:}\ c)$ if and only if (3.7) and (3.8) hold and

$$\lim_{n} (\tilde{a}_{nk} - \tilde{a}_{n,k+1}) \text{ exists.} \tag{3.15}$$

(iii) A = $(a_{nk}) \in (bs^u(T){:}\ c)$ if and only if (3.10), (3.11) and (3.12) hold and

$$\lim_{n\to\infty} \sum_k |(\tilde{a}_{nk} - (\tilde{a}_{k-1})| \text{ exists and} \tag{3.16}$$

$$\lim_{n} a_n \text{ exists.} \tag{3.17}$$

Corollary 3.9 (i) A = $(a_{nk}) \in (cs^u(T): c_0)$ if and only if (3.7)and (3.8) hold and

$$\lim_n \tilde{a}_{nk} = 0 \tag{3.18}$$

(ii) $A = (a_{nk}) \in (cs_o^u(T): c_0)$ *if and only if* (3.7) *and* (3.8) *hold and*

$$\lim_n (\tilde{a}_{nk} - \tilde{a}_{n,k+1}) = 0 \tag{3.19}$$

(iii) A = $(a_{nk}) \in (bs^u(T): c_0)$ if and only if (3.9) and (3.10) hold and

$$\lim_n \sum_k \left|(\tilde{a}_{nk}) - (\tilde{a}_{n,k+1})\right| = 0 \text{ and} \tag{3.20}$$

$$\lim_n a_n = 0 \tag{3.21}$$

Corollary 3.10 (i) A = $(a_{nk}) \in (cs^u(T): l_1)$ if and only if (3.7) holds and

$$\sum_k \left|\tilde{a}_{nk} - \tilde{a}_{n,k+1}\right| < \infty \tag{3.22}$$

$$\sup_{N,K \in \mathcal{F}} \left|\sum_{n \in N} \sum_{k \in K} (\tilde{a}_{nk} - \tilde{a}_{n,k-1}\right| < \infty \tag{3.23}$$

(ii) A = $(a_{nk}) \in (cs_o^u(T): l_1)$ if and only if (3.7) and (3.22)holds and

$$\sup_{N,K \in \mathcal{F}} \left|\sum_{n \in N} \sum_{k \in K} (\tilde{a}_{nk} - \tilde{a}_{n,k+1}\right| < \infty \tag{3.24}$$

(iii) A = $(a_{nk}) \in (bs^u(T): l_1)$ if and only if (3.9), (3.10) and (3.22), (3.24) hold and

$$\sum_n |a_n| < \infty \tag{3.25}$$

Corollary 3.11 (i) A = $(a_{nk}) \in (cs^u(T): l_p)$ if and only if (3.7) and (3.22) holds and

$$\sup_{k \in \mathcal{F}} \sum_n \left|\sum_{k \in K} \left(\tilde{a}_{nk} - \tilde{a}_{n,k-1}\right)\right|^p < \infty \tag{3.26}$$

(ii) A = $(a_{nk}) \in (cs_o^u(T): l_p)$ if and only if (3.7) and (3.22) holds and

$$\sup_{k \in \mathcal{F}} \sum_n \left|\sum_{k \in K} \left(\tilde{a}_{nk} - \tilde{a}_{n,k+1}\right)\right|^p < \infty \tag{3.27}$$

(iii) A = $(a_{nk}) \in (bs^u(T): l_p)$ if and only if (3.9), (3.10), (3.22) and (3.27) hold and

$$\sum_n |a_n|^p < \infty \tag{3.28}$$

The proof of corollary 3.8, corollary 3.9, corollary 3.10, corollary 3.11 are similar to theorem 3.7 therefore we omit their proofs.

References:

[1] B.Altay, F.Basar., On the paranormed Riesz sequence spaces of non-absolute type, Southeast Asian Bull Math. 26(2002), 701-715.

[2] B.Altay, F.Basar., Some paranormed Riesz sequence spaces of non-absolute type, Southeast Asian Bull Math. 30(2006), 591-608.

[3] F.Basar, B.Altay., Matrix mappings on the space bs(p) and its α-, β- and γ- duals, Aligarh Bull.Math. 21(1) (2002), 79-91.

[4] M.Basarir., On some new sequence spaces and related matrix transformations, Indian J. Pure and App. Math., 26(10)(1995), 1003-1010.

[5] B.Choudhary, S.K.Mishra., On Kothe-Toeplitz duals of certain sequence spaces and their matrix transformations, Indian J.Pure Appl. Math.24(5) (1993),291-301. 10

[6] B. Essakiammal @ Shunmugam, A. Pandiarani., On Some New Sequence Spaces of non - absolute type (Communicated)

[7] Ab.H. Ganie, N.A. Sheikh., On some new sequence spaces of non-absolute type and matrix transformations, J.Egyptian Math. Society, 21(2013),108-114.

[8] A.M. Jarrah, E. Malkowsky., Ordinary, Absolute and strong summability and Matrix transformations, FILOMATE 17(2003), 59-78.

[9] I.J. Maddox., Elements of Functional Analysis second ed., The Cambridge University Press, Cambridge, 1988.

[10] V.Karakaya, A.K.Noman,H.Polat., On paranormed λ-sequence spaces of non-absolute type, Math. Comput. Modelling 54(2011),1473-1480.

[11] E. Malkowsky, V. Rakocevic., On Matrix domains of triangles, App Math and Comp, 189(2007), 1146-1168.

[12] Meltam Kaya and Hasan Furkan., On the spaces of λ-convergent and bounded series(2013).

[13] M.Mursaleen, A.K.Noman., On some new difference sequence spaces of non-absolute type, Math. Comput. Modelling 52(3-4)(2010), 603-617. 11

[14] M.Mursaleen, A.K.Noman., On the spaces of λ-convergent and bounded sequences, Thai J. Math. 8(2) (2010), 311-329.

[15] S.Shanmugavel, A.Pandiarani., Some New classes of sequence spaces of non-absolute type, Int. J. Pure and App. Maths. 113(10)(2017) 119-128.

[16] A. Wilansky., Summability throught Functional Analysis, Vol.85 of North Holland Mathematics studies, North-Holland, Oxford, UK, 1984.